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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 89-0529
6a. NAME OF PERFORMING ORGANIZATION Rutgers The State University of N.J. — 6b. OFFICE SYMBOL (If applicable) N/A	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code) New Brunswick, N.J. 08903	7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR — 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-85-0247
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2304; TASK NO. A1; WORK UNIT NO.

11. TITLE (Include Security Classification)
REGULATION OF NONLINEAR AND GENERALIZED LINEAR SYSTEMS

12. PERSONAL AUTHOR(S)
Sontag, Eduardo D.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Final Technical Report	FROM 7-15-85 TO 7-14-88	9-14-88	

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

> Research during the grant period can be classified into these areas: Equilinearization, Sampling and Discrete Time Controllability, Input/Output Equations, Input/Output Stability, Families of Systems, Image Processing and Robotics, Nonlinear Stabilization, Computational Complexity in Control, and Neural Networks and Piecewise Linear Control. The report describes the progress achieved in each of these.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Mjr. James M. Crowley	22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5025 — 22c. OFFICE SYMBOL NM

DD FORM 1473, 83 APR — EDITION OF 1 JAN 73 IS OBSOLETE.

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REGULATION OF NONLINEAR AND GENERALIZED LINEAR SYSTEMS

Eduardo D. Sontag, AFOSR-85-0247

Final Scientific Report, September 1988

15 July 1985 to 14 July 1988

Research during the grant period can be classified into these areas:

1. Equilinearization
2. Sampling and Discrete Time Systems
   - Sampling
   - Discrete Time Controllability
3. Nonlinear System Theory
   - Input/Output Equations
   - Input/Output Stability
4. Families of Systems
5. Other topics
   - Image Processing and Robotics
   - Nonlinear Stabilization
   - Relations with Instantons
   - Computational Complexity in Control
   - Neural Networks and Piecewise Linear Control

The next sections describe the work in each of these.

1 Equilinearization

Early during the grant period we proved that a nonlinear controllable plant, under mild technical conditions, admits a precompensator with the following property: along control trajectories joining pairs of states, the composite system (precompensator plus plant) is, up to first order, a parallel connection of integrators. This relates to results of Rugh and of Champetier et. al. on pseudolinearization, but applies to linearizability along trajectories rather that around equilibrium points. A rather long paper in the IEEE Transactions in Automatic Control summarized the basic results ([6]). A closely related result in continuous-time system controllability insures that the conditions needed in that paper are in fact satisfied under very mild assumptions. This was explained in the published paper ([5]).

The above results are of interest in the context of the hierarchical approach to control typical in aerospace applications. There, it is often the case that open-loop trajectories are precomputed on the basis of optimal control or other considerations. Regulators are then tuned ("gain-scheduled") to the various operating conditions, or chosen for acceptable performance along the precalculated trajectories. On the basis of the above results, we have been able to provide a design method that allows for a 'universal' construction of regulators along families of desired trajectories. More recent research has dealt with concrete implementations using symbolic algebra packages, –explicit details of which are provided in the conference papers ([23], [24], [25]), where various aspects are analyzed,– and with the study of theoretical as well as practical performance of the methodology. At present we have a "computer-aided design method" (actually, a collection of MACSYMA routines) that carry out the necessary constructions. We showed by example the performance of our method in a problem dealing with the control of the angular velocity of a rotating satellite under the action of a single pair of opposing jets. The PI has lectured at various places, including NASA-AMES, Wisconsin, Florida, Purdue, and Cornell, as well as given various conference lectures, on this topic. Further, an invited hour lecture on this topic was presented at the Institute of Mathematics and Its Applications in Minneapolis, in July 1988, during the Signal Processing program's week on "Robust and Nonlinear Control with Aerospace Applications".

2 Sampling and Discrete Time Systems

Discrete-time control systems have been studied much less than their continuous counterparts, and their properties diverge considerably from those of the latter, due mainly to the possibility of singularities; this was to a great extent the main topic of the PI's doctoral dissertation 12 years ago.

2.1 Sampling

One major area of work relates to the topic of ***sampling***. When a continuous-time system is regulated by a digital computer, control decisions are often restricted to be taken at fixed times $0, \delta, 2\delta, \ldots$; one calls $\delta > 0$ the ***sampling time***. Under what is often called zero-th order hold sampled control, the resulting situation can be modeled through the constraint that the inputs applied be constant on intervals of length δ. It is thus of interest to characterize the preservation of basic system properties when the controls are so restricted. For controllability, this problem motivated the results in the classical paper of Kalman, Ho, and Narendra (1963). This studied the case of linear systems and established that controllability when sampling at frequency $1/\delta$ is preserved if $\delta(\lambda - \mu)$ is not of the form $2k\pi$ for any pair of distinct eigenvalues of the A matrix. In research carried out during the first year ([1]) we found a general result which in particular implies, for the large class of *bilinear* systems, an analogous property; one now needs that $\delta(\lambda + \lambda' - \mu - \mu')$ not be equal to $2k\pi$, k non zero, for any four eigenvalues of the autonomous dynamics matrix. Thus in the bilinear case, one must sample at 4 times (rather than twice) the natural frequencies of the system. The bilinear result is obtained by inducing a linear system on the adjoint representation of a certain Lie algebra associated to the given system. The result was proved in terms of the transitivity, often called also the "weak controllability," property. (Research under a previous grant had studied the more general non-bilinear case, and had resulted in general but not easily computable criteria.)

In recent work we have been able to extend this to the much more interesting ***accessibility*** property, with the same eigenvalue condition being sufficient. Details are provided in ([26]), and the treatment is based on the results about discrete-time systems described below.

Closely related to the above topic is another area of recent research by the PI, that of determining the effect of digital implementation on nonlinear control laws for continuous-time smooth systems. In particular, questions have been raised by J.Grizzle and others concerning the effect of such sampling on the currently very popular feedback linearization techniques.

The papers ([27], [14]) deal with this topic. We show there that the possibility of linearization via sampled control introduces certain strong constraints on the form of the controllability Lie algebra of the original system. In particular, we establish in those references that a system which is feedback linearizable via digital control must be a *graded* system in the sense of our work last year which dealt with the modeling of mechanical systems such as those arising in robotics. This result is obtained through the analysis of a set of necessary and sufficient conditions for sampled feedback linearization, conditions based on Lie-algebraic expansions of certain parametrized vector fields. We have benefited in this work from our interaction with B. Jakubczyk, a visitor at Rutgers during the period September 1986 to December 1987. (Jakubczyk was one of the coauthors of the paper, in 1980, which provided the first set of necessary and sufficient conditions for feedback linearization.)

2.2 Discrete Time Controllability

In the early 80's, a paper by B. Jakubczyk introduced the idea of studying *invertible* discrete nonlinear systems, and developed a realization theory which parallels much of the continuous time situation; further work along these lines was carried out in by Fliess, Normand-Cyrot, the PI, and others. Invertible systems are those for which transition maps, (one for each fixed control,) are all (local) diffeomorphisms. Invertibility is of course a priori an extremely strong assumption in the context of general discrete time systems. However, for systems that result from the sampling, as above, of continuous time systems, this assumption is always satisfied. For invertible discrete-time systems, it is possible to give a close analogue of the basic continuous-time orbit theorem of Stephan and Sussmann. Since times are discrete, it is of course not possible to take time derivatives with respect to time, as done classically. Instead, one substitutes derivations with respect to the values of the controls in each interval, the underlying assumption being that there is some sort of manifold structure on the control value set.

In a major paper during the first year and now published ([17]), we introduced a framework that allows us to prove an abstract orbit theorem, for "smooth actions on manifolds". This yielded as consequences both discrete-time and continuous-time controllability theorems. More interestingly, the theorem also implies a number of other results, including characterizations of "zero-time" orbits of various different types, and an alternative submanifold structure in the continuous-time case (the "input-topology" structure). Certain facts that would appear to be obvious, for instance the second countability of zero-time orbits (but not of arbitrary orbits), turned out to require careful proofs.

During this grant period we refined considerably these results, and are now able to provide very precise conditions for accessibility analogous to those available in the continuous-time case. This is also joint work with B.Jakubczyk, and a detailed paper ([9]) was submitted to *SIAM J. Control and Optimization*; the paper has been accepted subject to some revisions being carried out at this time. Also, we have extended and simplified the results in ([17]), and these will appear eventually as a book chapter in a volume based on a conference on Nonlinear Control that took place at Rutgers last year.

3 Nonlinear System Theory

We also studied various issues related to input/output behavior and its relation to state space properties.

3.1 Input/Output Equations

Starting with our work in the mid and late 70's, there have been many results relating the existence of i/o difference equations to finite realizability. These results, which provided analogues for nonlinear systems of the fact that a transfer function can be realized by a finite dimensional linear system if and only if it is rational, have been used by many authors in the context of identification problems. In ([10]) we showed how the simplest case, dealing with state-affine systems, has an analogue in the continuous time case. The proof is basically the same as in the older discrete case, but seems not to have been noticed in the currently increasing literature on these questions.

Specifically, by an (output-)*affine i/o equation* of order k we mean an equation of the type

$$\sum_{i=0}^{k} a_i(u(t), u'(t), \ldots, u^{(k-1)}(t))y^{(i)}(t) = b(u(t), u'(t), \ldots, u^{(k-1)}(t)), \tag{1}$$

where b and each a_i is a polynomial and a_k is not identically zero, satisfied by all pairs of smooth controls u and corresponding outputs y. The equation is (output-)*linear* if b is identically zero. For instance, transfer function descriptions of linear systems correspond in the time domain to affine i/o equations in which all the coefficients a_i are constants independent of u and b is linear. Precise definitions are given below. Note that in the *nonsingular* case when a_k is *always* nonzero, (1) implies finite realizability, since the highest derivative $y^{(k)}$ can then be expressed in terms of lower order derivatives of outputs and controls. However such "purely recursive" equations do not hold in general for nonlinear i/o maps; one of the main objectives of our discrete-time work was precisely the study of singular equations and how they relate to realizability. It is relatively trivial to show that such equations must exist if one assumes finite realizability, but they will in general not be affine in y; this is basically a transcendence degree argument. Bilinear realizability and a linear dimension argument do imply the existence of an affine equation, and our main results will provide a converse of this fact. The more general, non-bilinear case, will probably require techniques from algebraic geometry, as done for discrete-time in our thesis in 1976.

Roughly, the main results say that if an equation such as (1) holds for all i/o pairs arising from a (possibly unknown) system, then these are the i/o pairs of a finite dimensional smooth continuous time system, in fact a bilinear one. Technically, this hypothesis is stated in ([10]) in two different versions, one in terms of the existence of a locally convergent Volterra type of expansion and the other in terms of the existence of a nonlinear smooth (but not necessarily bilinear) realization.

3.2 Input/Output Stability

Constructions of coprime factorizations for nonlinear systems have been obtained of late in the literature, and the potential significance of such fraction representations to the theory of nonlinear control has been pointed out by many authors, in that they are of interest when studying the problem of parameterizing compensator laws. In ([11]), submitted during last Spring and already accepted for publication as a regular paper in the Transactions, we established that right factorizations exist under relatively weak hypotheses, and in doing so we make contact with the growing literature on nonlinear feedback control (see below). In order to develop the necessary techniques, we also had to provide what we believe are original definitions of input/output stability. These definitions refine those that had been typically used in operator theoretic approaches to nonlinear systems analysis and which were motivated by analogous linear concepts. Our definitions are more natural in the context of Lyapunov stability, and they may be relevant as well in areas other than the application to factorization problems.

Even for systems that are linearizable under feedback, it is not entirely clear that coprime factorizations should exist. This is because the construction of coprime factorizations is based on the use of feedback laws of the type

$$u = K(x) + v, \tag{2}$$

while in order to feedback-linearize systems one needs in general a state dependent term multiplying the control, such as

$$u = K(x) + \beta(x)v \tag{3}$$

with everywhere invertible but nonconstant β. Thus the intuition that "if a system is feedback linearizable then it must behave just as a linear system, and hence admit factorizations" is not *a priori* correct and requires careful analysis. We showed that indeed factorizations do exist in this case, however, but the argument is less trivial. In fact, we gave a general result which relates the existence of factorizations to the solution of ***smooth feedback stabilization problem(s).*** (See below.)

4 Families of Systems

This work continues the investigation of synthesis problems for parametrized families of systems. There are two main motivations for this line of research. The first is the expectation that parametrized controllers should prove useful in shifting the computational effort to offline preprocessing. This is useful in situations in which the precise values of some system parameters are not known in advance but can be determined on-line, as part of an indirect adaptive control scheme. The second motivation is more purely mathematical: it is natural to ask whether the constructions in control theory can be made "continuous" or "algebraic" in the system parameters.

Consider, for any fixed positive integers n,m, the set of all possible continuous-time systems

$$\dot{x}(t) = Ax(t) + Bu(t),$$

for A an $n \times n$ and B an $n \times m$ real matrix. We know that, if a given pair (A, B) is stabilizable, then there exists a feedback matrix $K = K(A, B)$ such that $A - BK$ is Hurwitz. This construction is continuous, in fact smooth, on the set of all stabilizable pairs (A, B), because a suitable $K(A, B)$ can be found via the solution of a well-posed quadratic optimization problem. What is not known is if a stabilizing $K(A, B)$ can be computed in a more algebraic fashion (the optimization argument depends on the implicit function theorem). We proved in the paper ([4]) that this can indeed be done provided that *dynamic* feedback be allowed (the precise definition of "algebraic" is via polynomial algebra). This result generalized a previous one by the author regarding stabilization of controllable systems. The proof in the noncontrollable case is completely different, however. In fact, in the controllable case the proof results in static, not dynamic, feedback; whether this can be done in the stabilizable case remains open.

Work in progress is related to pole assignment for parametrized families. We provided in ([3]) a necessary and sufficient condition for static parametric pole assignability, and started there research into the dynamic case. Joint work by the PI and a graduate student supported by this grant during the Summer of 87, showed that in certain cases a number of integrators linear in the plant dimension will suffice. The result is that $3n$ integrators will suffice for plants of dimension n provided that there be at most 3 independent parameters in the system description. The proof of the result ([13]) involves the study of vector bundles over the quaternions and generalized inverses of matrices over this division ring.

5 Other topics

A certain number of other topics where studied too.

5.1 Image Processing and Robotics

In ([20],[31]), we studied filtering problems for image segmentation, applying techniques of simulated annealing.

We obtained during the present grant period various results on the optimal control of robotic manipulators. This research, coauthored by H.Sussmann, applied modern algebraic techniques to the characterization of singular and optimal trajectories for two-link planar manipulators. All previous results in this area relied on numerical methods. We expect our results to provide a better conceptual understanding and to lead eventually to more efficient numerical methods. The conference papers ([19], [22]) reported on this work; the second of these is especially significant in that it emphasizes how a large number of results extend to the multilink manipulator case, and in fact to a large class of mechanical systems. Further work along these lines was presented as an invited hour talk at the AMS Summer School on Multibody Systems and Control organized by Marsden, Simo, and Krishnapasad in July 88. We showed how the structure responsible for the results is that of a particular type of hamiltonian system.

5.2 Nonlinear Stabilization

One of the central topics in mathematical control theory today is that of obtaining continuous or smooth stabilizers for nonlinear systems. It has been known for a long time, in part due to previous joint work of the author with H. Sussmann, that in general such stabilizers may not exist even if the original system is well-behaved, and that even when they do exist, there are major differences depending on whether dynamic of constant control is used (contrary to the situation in the linear case).

In ([15]) we proved that the angular velocity (Euler) equations for a rigid body can be smoothly stabilized with a single torque controller for bodies having an axis of symmetry. This complemented a recent result of Aeyels and Szafranski. Our proof was given in such a way as to generalize to a much larger class of systems, and is based on Center Manifold techniques. A general theorem on stabilization has resulted and a paper is being prepared on that topic.

5.3 Relations with Instantons

In a recent paper, U.Helmke showed how results of Donaldson in Yang-Mills theory are closely related to system theoretic notions, in particular to what are sometimes called "multirate systems". He then went on to provide a number of results on the topology of the space of framed instantons and of a certain space in which they can be naturally embedded. In ([7])

we pointed out that it is also possible to view in a natural way these same objects as bilinear systems, or equivalently, via minimal representations of matrix power series. An advantage of this alternative interpretation is that the machinery of Hankel matrices can then be applied to understand the structure of the corresponding moduli space. In particular, we obtained one natural representation of this quotient space as a quasi-affine variety. Explicit equations were given for the moduli space as a quasi-affine variety, using representation theory developed earlier by M.Fliess, the PI, and others.

5.4 Computational Complexity in Control

One of the most important and basic outstanding problems in control theory is that of finding necessary and sufficient conditions for deciding when a continuous-time analytic nonlinear system is (locally or globally) controllable. The goal is to provide some sort of generalization of the classical Kalman controllability rank condition. An early success of this line of research was achieved with the characterization of the *accessibility property*: there is a Lie-algebraic rank condition for deciding if it is possible to reach an open set from a given initial state. When this accessibility rank condition does not hold, all trajectories must remain in a lower-dimensional submanifold of the state space. It is known that local controllability can also be *in principle* checked in terms of linear relations between Lie brackets of the vector fields defining the system, and recent research has succeeded in isolating a number of necessary as well as a number of sufficient explicit conditions for controllability. No complete characterization is yet available, however.

The purpose of ([8]) was to point out that, whatever necessary and sufficient conditions are eventually found, these are likely to be rather hard to check. One way to quantify this difficulty is in terms of complexity of computation. The relative difficulty of controllability vis a vis the already understood accessibility problem is clarified in the case of the class of systems that can appear as subsystems of bilinear ones. This is a large class of nonlinear systems, including for instance all minimal realizations of finite Volterra series, as well as of course all linear systems. In the context of this class, one can make the precise statement that the accessibility question can be decided in polynomial time, while controllability is NP-hard. Work in progress by the PI attempts to provide tighter complexity estimates. The invited conference paper ([28]) shows decidability results for a more general class of systems.

5.5 Neural Networks and Piecewise Linear Control

The *backpropagation* algorithm has been proposed as a heuristic method for the acquisition of long term memory and pattern recognition in multilayered perceptrons. It is related to the perceptron learning algorithm, which does not generalize to the multilayered case. Experimental results have been highly encouraging, as described in a growing literature and in many recent conferences. The statement is typically made that the algorithm converges to a small number of possible local minima. Recently we started looking at this problem from the viewpoint of a nonlinear least squares procedure, and to analyze its behavior. It turns out that even for single layer perceptrons, for which it is often claimed that there is a unique local (hence global) minimum for the learning procedure, the picture is considerably more complicated. This means that the algorithms being used are not necessarily converging to the best selection of features for memory and recognition, and a more sophisticated learning rule is required. It relates naturally to the same question as it arises in our work on stochastic optimization via annealing (see [20]), since many neural networks can be seen as implementing various such optimization algorithms. The problems of local vs. global minima in both contexts are closely related. A recent paper gives an example of a problem with 125 training instances for which there is a false local minimum.

Our interest in this topic is through its connection to *piecewise linear systems*. It is widely acknowledged nowadays that there is a need to establish firm theoretical foundations for what is often called "logical control", since classical continuous methods are not adequate when dealing with large scale, complex systems in which computer control is involved. A few years back, we suggested a study of *piecewise linear (PL)* systems. These are essentially collections of linear systems under the supervisory control of finite automata. Such a model is in principle realistic in many applications, when different linear models are used about distinct operating points for a nonlinear system. The new idea was that of studying PL control systems through a *piecewise linear algebra* introduced by the PI. The perceptron work relates because the sets recognized by multilayer perceptrons with hard-limiting nonlinearities are the *same* as what we called piecewise linear sets in our older work. Thus the new heuristic learning algorithms may very well give rise to practical adaptive controllers for PL systems, controllers whose existence is guaranteed by our abstract results.

6 Remarks

Although not directly part of the grant effort, it is relevant to remark here that the PI has devoted a large amount of effort during this last two years to the launching of a new journal, *Mathematics of Control, Signals, and Systems (MCSS)*, published by Springer-Verlag. This is intended to be a leading publication in fields that include those in the scope of the present grant. As founder and co-managing editor (with Professor Bradley Dickinson of Princeton's Electrical Engineering Department), the PI expects MCSS to have a major influence in the future development of the area. A related activity was that of starting and co-editing (also with Dickinson) an electronic newsletter, distributed freely through various computer networks, on signal processing and control. The PI is also completing an introductory graduate level textbook in control theory.

The PI has been invited to a number of conference talks during the grant duration. In addition, he is one of the selected "topic speakers" at the next Conference on the Mathematical Theory of Networks and Systems (Amsterdam, June 89), and, as Vice Chair of the SIAM Activity Group in Systems and Control, is one of the organizers of the San Francisco Conference on "Control for the 90's" next Spring.

The PI was also selected as a member of the Panel that prepared the report on the "Future of Mathematical Control Theory," funded by NSF, AFOSR, and ONR.

7 Papers Appeared, Submitted, or in Preparation During Grant Period

7.1 Papers in Refereed Journals

1. "An eigenvalue condition for sampled weak controllability of bilinear systems," *Systems and Control Letters* 7(1986): 313-316.

2. "Review of *Multidimensional Systems Theory*," *Linear Alg. and Applications*, **87**(1987): 273-278.

3. "Comments on 'Some results on pole-placement and reachability'," *Systems and Control Letters* **8**(1986): 79-85.

4. "Continuous stabilizers and high-gain feedback," *IMA Journal of Mathematical Control and Information*, **3**(1986): 237-253.

5. "Finite dimensional open-loop control generators for nonlinear systems," *Int. J. Control* **47** (1988): 537-556.

6. "Controllability and linearized regulation," *IEEE Trans. Automatic Control* **32**(1987): 877-888.

7. "A remark on bilinear systems and moduli spaces of instantons", *Systems and Control Letters* **9** (1987): 361-368.

8. "Controllability is harder to decide than accessibility," *SIAM J. Control and Opt.*, **26**,5 (1988): to appear.

9. (With B. Jakubczyk) "Controllability of nonlinear discrete-time systems: A Lie-algebraic approach," submitted to *SIAM J. Control and Opt.*.

10. "Bilinear realizability is equivalent to existence of a singular affine differential i/o equation", *Systems and Control Letters*: to appear.

11. "Smooth stabilization implies coprime factorization," to appear in *IEEE Trans. Automatic Control*, 1989.

12. (with Y.Yamamoto) "On the existence of approximately coprime factorizations for retarded systems," submitted.

13. (with Y.Wang) "Pole shifting for families of linear systems depending on at most three parameters," to be submitted.

14. (with A. Arapostathis, B. Jakubczyk, H.-G. Lee, and S.I. Marcus), "The effect of sampling on linear equivalence and feedback linearization," submitted.

15. (with H.J. Sussmann), "Further comments on the stabilizability of the angular velocity of a rigid body," submitted.

16. (with H.J. Sussmann), "Backpropagation can give rise to spurious local minima even for networks without hidden layers," submitted.

7.2 Book Chapters

17. "Orbit theorems and sampling," in *Algebraic And Geometric Methods in Nonlinear Control Theory*, M.Fliess and M.Hazewinkel (Eds.), pp. 441-486, Reidel, Dordrecht, 1986.

18. "Reachability, observability, and realization of a class of discrete-time nonlinear systems," in *Encycl. of Systems and Control*, Pergamon Press, 1987, pp. 3288-3293.

7.3 Papers in Conference Proceedings

19. (With H. Sussmann) "Remarks on the time-optimal control of two-link manipulators," *Proc. IEEE Conf. Dec. and Control, 1985*, pp. 1646-1652.

20. (With H. Sussmann) "Image restoration and the annealing algorithm," *Proc. IEEE Conf. Dec. and Control, 1985*, pp. 768-773.

21. "Controllability and linearized regulation," *Proc. Conference Info. Sci. and Systems*, Princeton, 1986, pp.667-671.)

22. (With H.J.Sussmann) "Time-optimal control of manipulators," *Proc. IEEE Int.Conf.on Robotics and Automation*, San Francisco, April 1986, pp. 1692-1697.

23. "Equilinearization: A simplified derivation and experimental results," *Proc. Conf. Info. Sciences and Systems*, Johns Hopkins University Press, 1987, pp. 490-495.

24. "An explicit construction of the equilinearization controller", in *Proceedings of the 8th International Symposium on Mathematical Theory of Networks and Systems*, (C.I. Byrnes, C.F. Martin, and R. Saeks, eds.,) North Holland, Amsterdam, 1988.

25. "An approach to the automatic design of first-order controllers along reference trajectories," *Proc. IEEE Conf. Decision and Control*, Los Angeles, Dec.1987, pp.1363-1367.

26. "A Chow property for sampled bilinear systems," in *Proceedings of the 8th International Symposium on Mathematical Theory of Networks and Systems*, (C.I. Byrnes, C.F. Martin, and R. Saeks, eds.,) North Holland, Amsterdam, 1988.

27. (With B. Jakubczyk) "The effect of sampling on feedback linearization," *Proc. IEEE Conf. Decision and Control, Los Angeles, Dec.1987*, pp. 1374-1379.

28. "Some complexity questions regarding controllability," *Proc. IEEE Conf. Decision and Control, Austin, Dec. 1988.*

29. "Stabilizability, i/o stability, and coprime factorizations," *Proc. IEEE Conf. Decision and Control, Austin, Dec. 1988.*

7.4 Unpublished Reports

30. "Controllability is harder to decide than accessibility," Report RR No.24-87, *Rutgers Center for Operations Research*, July 1987.

31. (with H. Sussmann) "Optimization algorithms for image restoration and segmentation," Technical Report No.34, *Rutgers Center for Computer Aids for Industrial Productivity*, April 1987.

32. "Some remarks on the backpropagation algorithm for neural net learning," Report SYCON-88-02, *Rutgers Center for Systems and Control*, June 1988.

33. "Integrability of certain distributions associated to actions on manifolds and an introduction to Lie-algebraic control," Report SYCON-88-04, *Rutgers Center for Systems and Control*, July 1988.